Title: A Twist on Goldbach's Conjecture

Link to Outcomes:

- **Problem Solving** Students will alter one or more of the critical attributes of Goldbach's Conjecture in order to create original mathematical conjectures.

- **Communication** Students will support conjectures in writing.

- **Reasoning** Students will investigate the meaning of Goldbach's Conjecture through data collection and analysis.

- **Arithmetic Operations** Students will utilize the appropriate arithmetic operation for a proposed conjecture.

- **Number Relationships** Students will explore number theory as it relates to mathematical hypotheses.

- **Patterns/ Relationships** Students will analyze data based on pattern recognition.

Brief Overview:

Together, the class will explore Goldbach's Conjecture. With guidance, each student will formulate a variation of Goldbach's Conjecture. Individually, each student will collect data related to this conjecture and write a conclusion discussing any patterns and observations.

Grade/Level:

Grades 6-8

Duration/Length:

Three class periods should be allocated. Students turn in a finished product on day four.

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- Applying divisibility rules
- Identifying prime and composite numbers
- Organizing data
- Constructing tables
- Recognizing patterns

Objectives:

- Students will investigate Goldbach's Conjecture.
- Students will formulate conjectures.
- Students will identify and describe numerical patterns.
- Students will write conclusions based on data analysis.

Materials/Resources/Printed Materials:

- paper
- pencils
- pens
- magnetic sentence strips listing the attributes of Goldbach's Conjecture:
  every, even, number, greater than, two, can be written as, the sum of, two, primes
- Worksheet: A "What If Not..." for Goldbach's Conjecture
- Worksheet: Rubric for A "What If Not..." for Goldbach's Conjecture

Development/Procedures:

Day 1: Introduction

- Check for understanding of prerequisite knowledge.
  Review and discuss characteristics of prime and composite numbers to distinguish between the two sets.

- Introduce Goldbach's Conjecture:
  Every even number greater than two can be written as the sum of two primes.
  Discuss the meaning of Goldbach's Conjecture.
  Generate a list of examples to support this conjecture.
  Ask students if an even number greater than two exists that can **not** be written as the sum of two primes.

- Identify the separate attributes of Goldbach's Conjecture.
  <u>every</u> <u>even</u> <u>number</u> <u>greater than</u> <u>two</u> <u>can be written as</u>
  <u>the sum of</u> <u>two</u> <u>primes</u>
  Place magnetic sentence strips listing the attributes of Goldbach's Conjecture on the chalkboard, allowing students to manipulate them.
  Discuss how the critical attributes can be changed:
  > *even* may be changed to *odd*, *prime*, or *composite*
  > *two* may be changed to *zero*, *one*, *three*, or *four*, etc.
  > *the sum of* may be changed to *difference of*, *product of*, or *quotient of*
  > *primes* may be changed to *composites*

- Elicit a sample student conjecture.
  Distribute "A 'What If Not...' for Goldbach's Conjecture" task worksheet.
  Model the development of the task, highlighting all four sections.
  Distribute Rubric for "A 'What If Not...' for Goldbach's Conjecture."
  Address rubric criteria.

- Instruct students to formulate a conjecture that will be checked on Day Two.

Day 2: Conjecture Proposal

- Instruct students to work on task development.

- Conduct student-to-teacher mini-conference in order to verify the validity of student's conjecture statement.

- Once student conjecture has met teacher approval, students continue task completion.

Day 3: Completion of Conjecture Task

• Completion of rough draft takes place during class period in order for student and peer evaluation to occur.

• Inform students final copy is due at the beginning of Day Four.
     Option: If a student has not completed peer evaluation in class, a parent may be used in place of peer for evaluation purposes.

Day 4: Collection of Conjecture Task

• Collect final copy of student's "A 'What If Not...' for Goldbach's Conjecture" with Rubric stapled on top.

Evaluation:

During the introduction of the lesson, the teacher will informally assess student understanding of prerequisite knowledge and attribute identification. The teacher uses student responses to develop the modeling of the "What If Not..." task. Through student-to-teacher mini-conferences, teacher evaluates the workability of each proposed conjecture. Teacher circulates through the classroom in order to monitor student progress. Students will take part in self and peer evaluation procedures. When scoring the final student conjecture, teacher utilizes rubric.

Extension/Follow Up:

Game based upon Goldbach's Conjecture:
In a group, player one chooses an even number between 10 and 100 and announces it to the group. Other players race to find two prime numbers that add up to the given number. To win, the player who finds the two prime addends first earns one point and takes player one's place naming the next number.

References:

Anne Arundel County Public Schools. *Anne Arundel County Curriculum, Format # 3*. Grade 6, p. 68, 1993.

Brown, Steven I. and Marion I. Walter. *The Art of Problem Posing*. Franklin Institute Press, 1983.

Stempien, Margaret, Ph.D.   Graduate course in Number Theory. Indiana University of Pennsylvania.

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A "What If Not..." for Goldbach's Conjecture

Goldbach's Conjecture:
　　*Every **even** number greater than **two** can be written as the **sum** of **two** primes.*

Critical Attributes of Goldbach's Conjecture:
　　"even," "two," "sum," "two," "primes"

TASK:

The title of this assignment is _____'s Conjecture. The paper may be written in pencil or pen, or it may be typewritten. The paper will contain four sections; each section should be labeled appropriately.

•Conjecture
　State your conjecture. You must change at least one attribute from Goldbach's Conjecture.

•Changes to Goldbach's Conjecture
　Explain which attributes you have changed and your reason(s) for making these changes.

•Data
　List at least fifteen examples to support your conjecture. Present the data in two columns. Each column must be titled. For example:

even number greater than 2	two prime addends
4	2 + 2
6	3 + 3
8	3 + 5

　The first ten examples should be for consecutive numbers. Also include five "way out" examples.

•Conclusion
　Write at least ten sentences in which you discuss your observations from investigating your conjecture, describe any patterns you noticed in your data, tell whether or not you believe your conjecture is true, and share any new ideas which occurred to you during the course of this assignment.

Rubric
for
A "What If Not..." for Goldbach's Conjecture

EVALUATION CRITERIA	POINTS	STUDENT	PEER/ PARENT	TEACHER
following directions				
.....title	1			
.....heading	1			
.....margins	1			
.....labeled sections	1			
neatness	3			
content and written expression				
.....conjecture	1			
.....changes made to Goldbach's Conjecture	3			
.....data	5			
.....conclusion	10			
spelling	4			
..................total possible	30			